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AN INVESTIGATION OF THE RELATIONSHIPS BETWEEN MOUNTAIN-WAVE CONDITIONS AND CLEAR AIR TURBULENCE ENCOUNTERED BY THE XB-70 AIRPLANE IN THE STRATOSPHERE

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16. Abstract

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SUMMARY

The data from 36 XB-70 flights conducted over the mountainous regions of the western United States together with rawinsonde data were used to investigate relationships between conditions favorable for mountain waves and clear air turbulence. Profiles for the Scorer parameter and the gradient Richardson number were evaluated from an eight-level model. The Scorer parameter and the gradient Richardson number profiles were computed from the rawinsonde data on those days when the XB-70 flew, and these results compared to the model profiles and related to the reported turbulence. Ascent rate profiles of rawinsonde balloons were analyzed from which the presence of mountain or lee waves was inferred. From the results of this investigation, objective methods were developed for forecasting the occurrence or non-occurrence of turbulence in the stratosphere due to mountain waves.

INTRODUCTION

The problem of stratospheric turbulence was vividly brought to light when the supersonic XB-70 airplane made numerous test flights in the western United States during 1965 and 1966 (ref. 1). With aviation on the threshold of routine supersonic flight in the stratosphere, further investigation of this phenomenon is justified on the basis of safety and passenger comfort.

Extensive analyses have been accomplished by Scoggins (ref. 2) to determine the synoptic features of the stratosphere when the XB-70 encountered turbulence. Several indices and relationships were examined to test their reliability as turbulence indicators.

Ehernberger (ref. 3) suggests that events in the troposphere and possibly mountain waves may be related to the occurrence of turbulence in the stratosphere. However, conditions in the troposphere have not been analyzed to determine if such a relationship exists.

Mitchell and Prophet (ref. 4) suggest that wave motions in the stratosphere can lead to turbulence. If wave motions occur, the lapse rate of temperature over a small layer can become very large, and such a condition is conducive to turbulence. Additionally, Ashburn et al. (ref. 5) have reported that the highest frequency of turbulence in the stratosphere occurs during the winter. This is the time of year when mountain wave activity would reach a peak over the western United States because of the seasonal intensification of the westerly winds.

Foltz (ref. 6) has shown that lee waves may lead to clear air turbulence (CAT), and that the two are associated. In addition, he has shown that the degree of severity of CAT is a function of the amplitude and wavelength of the lee waves. On the basis of Foltz' results, it will be assumed that conditions in the troposphere that are favorable for lee waves also will be favorable for CAT.

The data from XB-70 flights (ref. 1) conducted over the mountainous regions of the western United States together with rawinsonde data were used to investigate relationships between mountain-wave conditions and CAT. The theory of the development of mountain waves is discussed as well as the general causes of atmospheric turbulence. Certain aspects of each phenomenon are investigated.

Other theoretical aspects of the problem are discussed in order to establish a firm and reasonable foundation for the hypothesis that the development of mountain waves in the troposphere can lead to significant turbulence in the stratosphere. These theoretical considerations include the vertical propagation of a wave and wave energy, and the breakup of waves into turbulent eddies.

The investigation of this problem is limited to the synoptic scale since this is the only type of meteorological data readily available to accompany the reported turbulence. Routine techniques are used to analyze the synoptic data. Also, the measured meteorological data are used to evaluate specific theoretical equations which have been developed from mountain-wave and turbulence theory.

An additional objective of this research is the development of reliable and simple forecast methods to predict the occurrence of stratospheric turbulence due to mountain waves. These forecasting techniques and procedures, based solely on the use of routine meteorological data, are developed for use in the weather station and/or forecast center.

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BACKGROUND OF THE PROBLEM

The Theory of Mountain Waves

Scorer (ref. 7) analyzed the problem of airflow over mountains. His theoretical development is valid for dry, streamline flow which is also inviscid and isentropic. Scorer gives the equation for the stream function of a disturbance as

$$\psi'' - \left(\frac{g}{c^2} + \beta \right) \psi' + (L^2 - k^2)\psi = 0 \tag{1}$$

where

$$L^2 \equiv \frac{g\beta}{U^2} - \frac{U''}{U} \tag{2}$$

and

$$\beta \equiv \frac{1}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z} \tag{3}$$

In the above equations, L^2 is the Scorer parameter, β is a stability factor, ψ is the stream function, a prime represents differentiation with respect to height, g is the acceleration due to gravity, c is the speed of sound, k is the wave number, U is the horizontal wind speed, U'' is the derivative with respect to height of the vertical wind shear, $\bar{\theta}$ is the mean potential temperature of a layer, and z is the height. An order-of-magnitude analysis for equation (1) shows that the $(L^2 - k^2)$- term is the most important in the solution, while the term $(gc^{-2} + \beta)$ is negligible.

Scorer's analysis of airflow over mountains highlights the flow features in the lower troposphere and particularly the lee-wave aspects at or near mountain-top level. However, this is only one aspect of the total motion, and Scorer suggests the possibility of a relationship between airflow over mountains and stratospheric mother-of-pearl clouds. Further investigation by Scorer (ref. 8) indicates that the maximum amplitude of a lee wave is found near the maximum value of L^2, and that lee waves exist only if L^2 decreases with height. If wave motions develop by orographic means, Scorer contends that the stratosphere is capable of propagating these waves in the vertical direction because, with height, there is a rapid increase of potential temperature and decrease of wind velocity.

Other researchers used Scorer's (refs. 7 and 8) theory of airflow over mountains to determine streamline patterns at great heights in the atmosphere. Corby and Sawyer (ref. 9) conclude that some mountain waves have a maximum amplification in the upper troposphere and stratosphere depending on the static stability of the atmosphere near the tropopause. Palm and Foldvik (ref. 10) show that with given wind and temperature profiles, harmonic analysis yields a multitude of possible wavelengths for airflow over mountains. The interaction of the large wavelengths results in 40-km wavelengths at stratospheric heights, which are compatible with the observed features of mother-of-pearl clouds. Additional analysis reveals that a mountain 700 m high is capable of causing a vertical displacement of 400 m in the airstream at stratospheric heights.

Mountain Waves and Clear Air Turbulence (CAT)

Gazzola (ref. 11) summarizes atmospheric conditions favorable for the development of gravity waves resulting from orographic effects. Variables include terrain features, temperature stratification, and the distribution in the vertical of the horizontal wind. A temperature stratification favorable for the development of wave motions includes: 1) A layer of low stability near the ground, 2) a very stable layer above the surface layer, and 3) a low stability layer above the very stable layer. A vertical wind profile favorable for wave development contains: 1) A component of wind normal to the crest of the mountains with a minimum magnitude between 7 and 15 m sec^{-1}, 2) little variation of the wind direction with height, and 3) a gradual increase of wind speed with height into the upper troposphere. These conditions occur frequently when a properly oriented jet stream or zone of maximum wind moves across a mountain chain.

Turbulence in the atmosphere is produced and dissipated through the actions of buoyancy and shearing stress. The Richardson number, Ri, is defined as the ratio of buoyancy to the square of vertical wind shear. The gradient Richardson number is written (ref. 12) as

$$\mathrm{Ri} = \frac{\frac{g}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z}}{\left(\left| \frac{\partial \vec{\bar{V}}}{\partial z} \right| \right)^2} \tag{4}$$

where $(\partial \vec{\bar{V}}/\partial z)$ is the mean vertical vector wind shear, and the remaining parameters are the same as defined for the previous equations. Under most atmospheric conditions, turbulence is created by shear and destroyed by buoyancy. Therefore, it would be reasonable to expect the magnitude of the Richardson number to show some positive relationships to atmospheric turbulence.

The relationship of Ri to CAT does not, however, always yield conclusive results. Veazey (ref. 13) summarized numerous attempts to relate Ri and CAT, but problems including poor data resolution, scale length, non-synchronous data collection, and the sign of the vertical wind shear were serious enough to produce a wide variety of results. Nevertheless, Scoggins (ref. 2) analyzed the synoptic conditions in the stratosphere associated with XB-70 turbulence and found Ri to be the best of the turbulence indicators evaluated. However, Ri was not an effective index for categorizing non-turbulent conditions.

In determining the relationships between CAT and lee waves, Foltz (ref. 6) states that terrain, wind speed, wind shear, and thermal stability are important parameters to investigate. Foltz qualifies the importance of the wind shear term in equation (2) by limiting its significance to the surface boundary layer and sometimes near the jet stream.

If wave motions which are produced orographically reach the stratosphere, what causes their breakup into eddies? Axford (ref. 14) offers two explanations for the breakup of wave motions into turbulence. The first is a local change in the Richardson number. The second suggests that a downward vertical velocity leads to an overturning of the wavecrest (Kelvin-Helmholtz instability) resulting in cold air sinking through a warm environment. Scorer (ref. 15) shows that a local decrease in the Richardson number occurs when a streamline undergoes a positive vertical displacement. Badgley (ref. 16) indicates that downward, large-scale vertical motion is capable of reducing the value of Ri by as much as one-half in a matter of three to six hours. Scoggins (ref. 2) shows that fields of vertical motion of either sign on a synoptic scale in the stratosphere are highly correlated to the turbulence encountered by the XB-70 airplane. Thus, downward motion in the wave may lead to overturning of the wavecrest and turbulence, while the upward motion in the wave may help it to grow, become unstable, and break up into eddies.

The Vertical Propagation and Transfer of Energy of Mountain Waves into the Stratosphere

Hines (ref. 17) shows that wave motions which develop in the troposphere are capable of propagating vertically to great heights in the atmosphere. Typical wave dimensions that result from airflow over mountains (ref. 10) are capable of propagating vertically to heights well above the stratosphere (above 50 km). However, Eliassen and Palm (ref. 18) show that mountain waves cannot readily propagate vertically into the stratosphere unless their wavelengths are greater than about 26 km. In fact, they state that wavelengths less than 17 km undergo total reflection within the troposphere.

An important aspect of this problem concerns the vertical propagation of the wave and wave energy under the influence of wind shear (ref. 19). A critical level, if it exists, is the level at which the horizontal phase velocity of the wave equals the mean wind speed. If a wave passes through a critical level, the Reynolds stress, which is a measure of wave magnitude, decreases according to

$$\exp \{ - 2\pi (R_c - \tfrac{1}{4})^{\frac{1}{2}} \} \tag{5}$$

where R_c is the Richardson number at the critical level. Thus, a wave traveling through a critical level is strongly attenuated if R_c is large. According to Booker and Bretherton (ref. 19), at the critical level the wave transfers its momentum to the mean flow and dissipates. While these investigators did not consider viscosity and heat conduction as causes of wave absorption, Hines and Reddy (ref. 20) studied the effect of wind shear as well as viscosity on wave motions and arrived at the opposite conclusions. According to Hines and Reddy, reflection of a wave rather than absorption occurs at the critical level, and there is no momentum transfer to the mean flow.

Another matter for consideration is the manner in which the transfer of wave energy takes place. Eliassen and Palm (ref. 18) hypothesize that waves extract energy from the mean flow at certain levels and deposit it at others. Hines and Reddy (ref. 20) state that the wave transmits energy horizontally across the sloped surface of the perturbed interface through the action of a vertical flux, and thus the energy enters the layer above. The precise mechanism for the transfer of wave energy vertically through the atmosphere remains unclear.

The theoretical discussions of wave propagation are complex and sometimes conflicting. However, the general conclusion that wave motions originating in the troposphere can propagate vertically to great heights in the atmosphere encourages further investigation of mountain waves and their relationship to stratospheric turbulence.

The Influence of Wind and Stability on Profiles of the Scorer Parameter and the Richardson Number

From a theoretical viewpoint, L^2 and Ri are important parameters to investigate for determining whether any relationship exists between mountain waves and CAT. From analytical considerations, they are easy to calculate. Variations in these parameters under atmospheric conditions favorable and unfavorable for mountain waves are of vital interest to this research.

An eight-level model was utilized in the evaluation of these parameters. The standard pressure levels for the model were selected in an effort to obtain layers of nearly equal thickness while still using those pressure levels that are available normally for routine analyses.

Based upon observed and theoretical conditions for the existence of mountain waves, two wind profiles were synthesized, the first having strong wind speeds and large vertical shears, while the second had light wind speeds and small vertical shears. In addition, three temperature profiles were synthesized. The first was obtained from the 1962 U. S. Standard Atmosphere and used as a reference to develop two other temperature profiles. The second profile had a lapse rate of temperature between all levels that was less than the corresponding lapse rate of temperature for the Standard Atmosphere. In this study, this profile is referred to as a relatively stable temperature profile. Conversely, the third temperature profile is relatively unstable because its lapse rate between all the selected levels is greater than that for the Standard Atmosphere with one exception. This exception was necessary to maintain realistic temperature values near the tropopause. In effect, the relatively unstable temperature profile displays a lower tropopause than the other temperature profiles. For the corresponding pressure levels, identical wind directions were used for all wind profiles, and the heights selected for the pressure levels were those of the Standard Atmosphere. Therefore, variations among the values of L^2 and Ri for these wind and temperature combinations will be due solely to the effects of wind speed, wind shear, and stability. The complete data for the model profiles of wind and temperature are shown in table I.

TABLE I

PROFILES OF WIND AND TEMPERATURE FOR MODEL ATMOSPHERES

Level, mb	Standard atmosphere height, m	Wind			Temperature, °C		
		Direction, degrees	Speed, m sec^{-1}		Relatively stable	Standard atmosphere	Relatively unstable
			Strong	Light			
850	1457	240	5	4	6.0	5.5	5.0
700	3012	260	12	8	-2.0	-4.7	-6.0
500	5574	265	20	12	-17.0	-21.0	-26.0
400	7185	270	30	18	-27.0	-31.8	-40.0
300	9164	265	42	23	-39.5	-44.7	-57.0
200	11 748	270	25	16	-51.0	-56.6	-60.0
150	13 608	275	15	11	-50.5	-56.6	-64.0
100	16 180	280	7	7	-50.0	-56.6	-67.0

A computer program was prepared for the IBM 360/65 to handle the calculations of L^2 and Ri for these atmospheric models. The computer program was designed to perform the intermediate calculations necessary to convert the profile data into values of the proper units required for the evaluation of equations (2) and (4). The method of finite differences was used to evaluate the partial derivatives in these equations.

Each wind profile was used in combination with each of the three temperature profiles, and the L^2 and Ri values were calculated for the appropriate atmospheric layers. The profiles of L^2 are shown in figure 1 for the strong wind data in combination with each temperature profile, and in figure 2 for the light wind data in combination with each temperature profile. For each profile

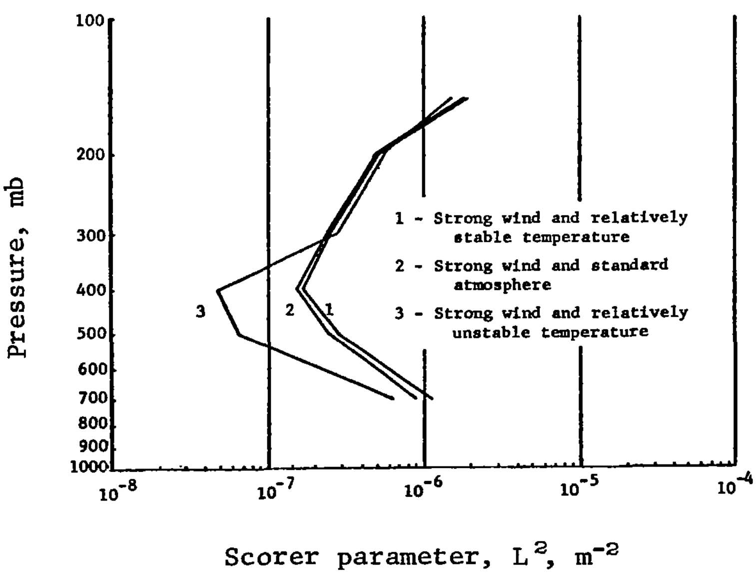


Figure 1.- The profiles of L^2 for the strong wind profile in combination with selected temperature profiles.

in figure 1, there is a decrease of about one order of magnitude or more in the value of L^2 from the 700-mb level to the minimum point of the profile at the 400-mb level. In figure 2, the profile of L^2 for the combination of the light wind and relatively unstable temperature data exhibits a similar result. However, the L^2 profiles for the light wind condition in combination with the Standard Atmosphere and relatively stable temperature profile do not have as great a slope from 700 mb to the minimum point at 400 mb. Also, the magnitude of L^2 at all levels below 300 mb in these latter cases is definitely larger than the former cases. For all these profiles, the value of L^2 increases steadily above 300 mb.

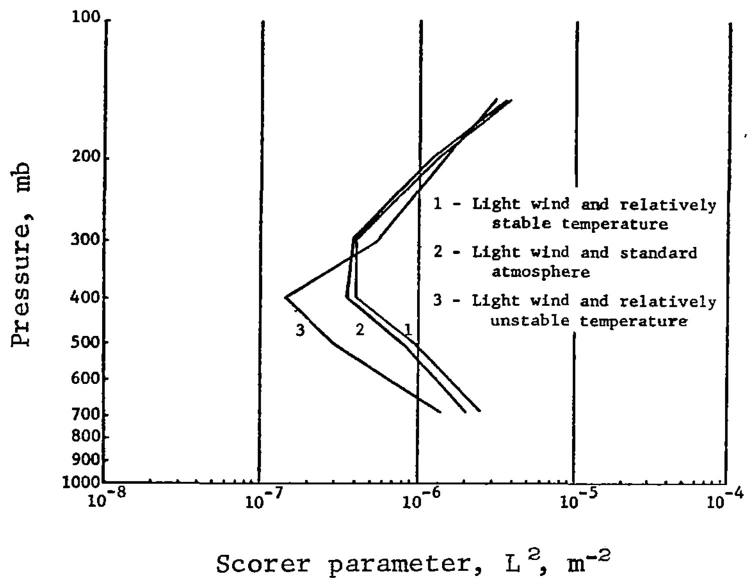
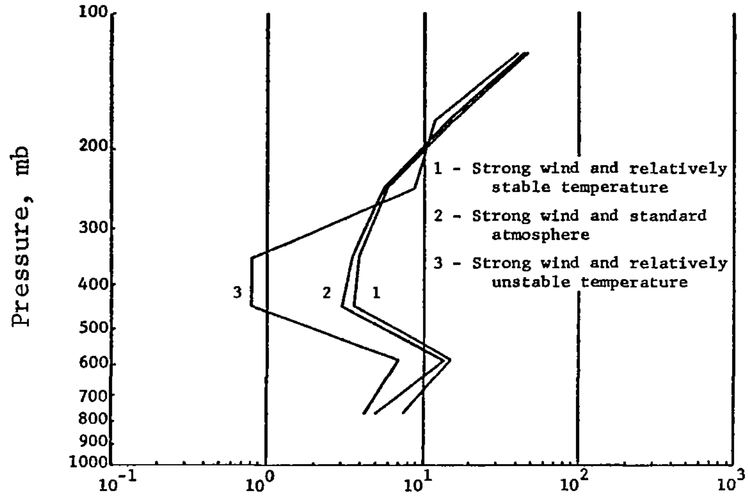


Figure 2.- The profiles of L^2 for the light wind profile in combination with selected temperature profiles.

The profiles of L^2 in figures 1 and 2 differ somewhat from previously published profiles (ref. 10). The atmospheric models show that L^2 decreases from 700 to 400 mb, while other published profiles show the decrease to about the 200-mb level. Because of the limited number of levels in the model, important features of the wind and temperature profiles undergo considerable smoothing. These features include the tropopause height, the height and speed of the maximum wind, and the wind shear. Also, stratospheric data enter into the computation of the L^2 values at the 300-mb level. Since θ increases rapidly in the stratosphere, there will be a marked increase in the first term of equation (2) beginning at the 300-mb level. The second term of the same equation yields a negative value at 300 mb due to changes in wind shear with height; this causes an increase in L^2 at that level. Also, above the level of the maximum winds, the denominator of each term in equation (2) decreases, thereby leading to an increase in each term in the equation. If more pressure levels had been included in the model, the profiles of L^2 would have been more compatible with previous examples.

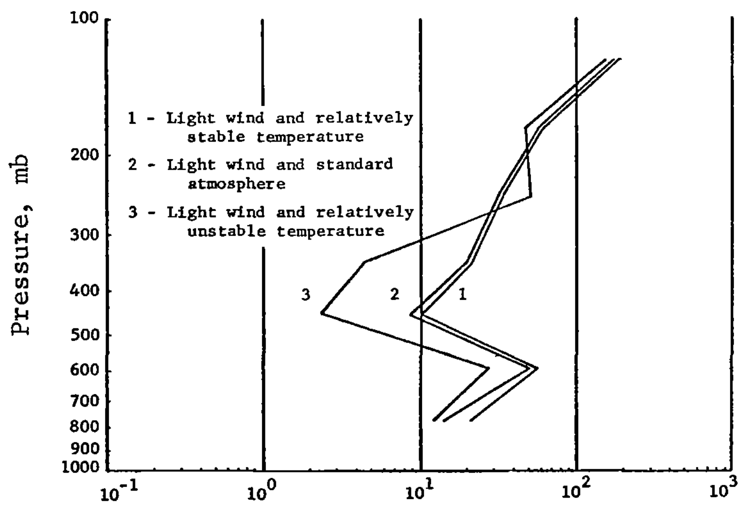
As shown in figure 3, the profiles of Ri with strong winds display an increasing value of Ri from about 850 to above 700 mb, and then a decreasing value to about 400 mb. Above the 400-mb level, there is a gradual increase in the value of Ri. The magnitude of Ri for all the strong wind cases is generally



Richardson number, Ri

Figure 3.- The profiles of Ri for the strong wind profile in combination with selected temperature profiles.

less than 10 below the 200-mb level. In figure 4, the profiles of Ri for the light wind data in combination with the temperature from the Standard Atmosphere and the relatively stable temperature profile exhibit the same shape as



Richardson number, Ri

Figure 4.- The profiles of Ri for the light wind profile in combination with selected temperature profiles.

the strong wind cases, but the magnitude of Ri is much larger and generally greater than 10. For the case of the light wind and relatively unstable temperature data, an intermediate result occurs for the Ri profile. While the general shape of the Ri profile is the same as in the strong wind cases, the magnitude of Ri is greater than 10 below 500 and above 300 mb, and less than 10 between these levels.

All the strong wind profiles plus the combination of the light wind and relatively unstable temperature data lead to a large decrease of L^2 with height from the top of the mountains to the middle troposphere, and to low values of Ri. These conditions are favorable for the development of mountain waves and the occurrence of turbulence. The remaining light wind and temperature combinations show a small decrease of L^2 with height and large values of Ri, but these conditions are unfavorable for mountain waves and turbulence. Therefore, the profiles of L^2 and Ri for the models are classified as characteristic and non-characteristic of the development of mountain waves and the occurrence of turbulence.

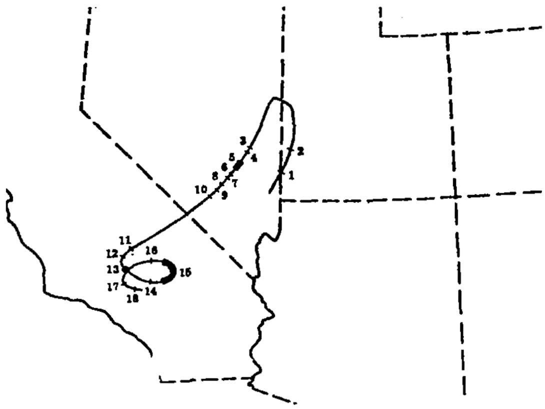
It is of interest to note the significance of the shear term in equation (2) for the various combinations of wind and temperature data. The terms in equation (2) were computed separately, and the magnitude of the ratio of the first to second term was calculated. In all the strong wind cases where the profiles of L^2 were favorable for mountain waves, there was at least one level where the ratio of the terms was 1:1 or less. For the light wind and relatively unstable temperature data, the minimum ratio was 2.7 : 1, while for the remaining light wind and temperature combinations, the lowest ratio was 4:1. Thus, it appears that large wind shears from mountain-top level to the tropopause may play a significant role in the development of mountain waves.

ANALYSES OF DATA

Grouping of Data

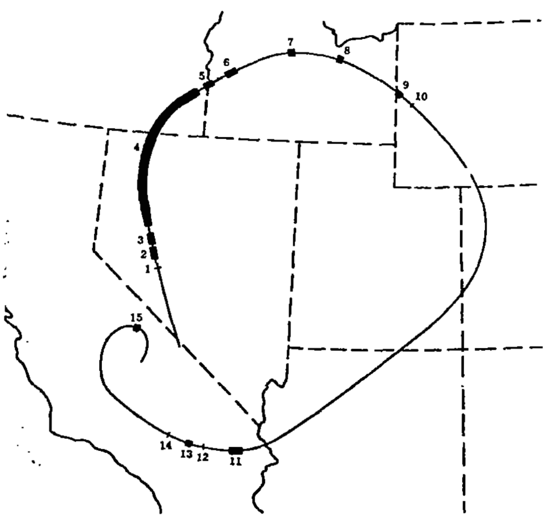
Aircraft.- The turbulence encountered by the XB-70 aircraft was reported in reference 1. The data presentation included the flight track of the airplane and the time, altitude, location, and extent of the turbulence encounters. These encounters were determined from changes in the normal acceleration at the center of gravity of the aircraft. The changes were measured by a recorder on the aircraft. The flight levels for these encounters ranged from 12 to 22 km. Examples of these data are shown in figure 5.

In the 36 cases presented by Ehernberger, each flight had at least one encounter with turbulent conditions. In this research, a flight with five or less instances of reported turbulence along the flight track is considered as a no-turbulence case while a flight with six or more turbulent encounters is considered as a turbulence case. These criteria represent a division within the data since all flights classified as turbulence cases had at least eight turbulence encounters along the flight path, versus the five or less encounters for the no-turbulence cases. The results of this classification showed 17 cases of turbulence and 19 cases of no turbulence. Future references to turbulence and no-turbulence cases will be based on this classification unless otherwise indicated.



Encounter number	Greenwich time	$\Delta a_{n_{max}}$, g units	Distance in turbulence, n. mi. (km)	h_p, ft (m)
1	1845Z	0.25	2.1 (3.9)	40.7×10^3 (12.4×10^3)
2	1849Z	.15	4.9 (9.1)	43.1 (13.1)
3	1901Z	.15	4.1 (7.6)	43.8 (13.4)
4	1901Z	.20	2.5 (4.6)	44.5 (13.6)
5	1902Z	.20	14.8 (27.4)	45.1 (13.7)
6	1903Z	.20	2.1 (3.9)	45.3 (13.8)
7	1903Z	.15	.8 (1.5)	45.5 (13.9)
8	1904Z	.15	4.7 (8.7)	45.2 (13.8)
9	1906Z	.15	1.9 (3.5)	44.2 (13.5)
10	1907Z	.20	2.2 (4.1)	46.0 (14.0)
11	1915Z	.15	2.9 (5.4)	49.8 (15.2)
12	1915Z	.20	1.4 (2.6)	49.8 (15.2)
13	1919Z	.20	5.1 (9.4)	49.8 (15.2)
14	1920Z	.15	2.7 (5.0)	49.6 (15.1)
15	1926Z	.45	56.2 (104.1)	42.0 (12.8)
16	1928Z	.15	1.5 (2.8)	41.3 (12.6)
17	1931Z	.15	1.4 (2.6)	40.3 (12.3)
18	1933Z	.15	1.8 (3.3)	41.0 (12.5)

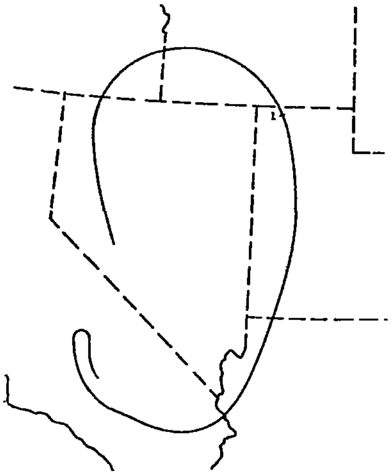
(a) 4 March 1965



Encounter number	Greenwich time	$\Delta a_{n_{max}}$, g units	Distance in turbulence, n. mi. (km)	h_p, ft (m)
1	1905Z	0.10	3.4 (6.3)	51.0×10^3 (15.5×10^3)
2	1906Z	.70	20.4 (37.8)	52.0 (15.8)
3	1908Z	.45	13.9 (25.7)	53.6 (16.3)
4	1913Z	1.20	242.9 (449.9)	57.0 (17.4)
5	1919Z	.25	12.1 (22.4)	62.2 (19.0)
6	1921Z	.30	17.2 (31.9)	62.8 (19.1)
7	1923Z	.20	10.2 (18.9)	64.3 (19.6)
8	1925Z	.15	10.7 (19.8)	65.0 (19.8)
9	1930Z	.15	10.0 (18.5)	65.6 (20.0)
10	1931Z	.10	3.1 (5.7)	65.8 (20.1)
11	1958Z	.20	19.7 (36.5)	69.4 (21.2)
12	2000Z	.10	4.2 (7.8)	70.4 (21.5)
13	2001Z	.20	6.9 (12.8)	70.0 (21.3)
14	2003Z	.20	4.5 (8.3)	69.4 (21.2)
15	2015Z	.10	5.6 (10.4)	53.2 (16.2)

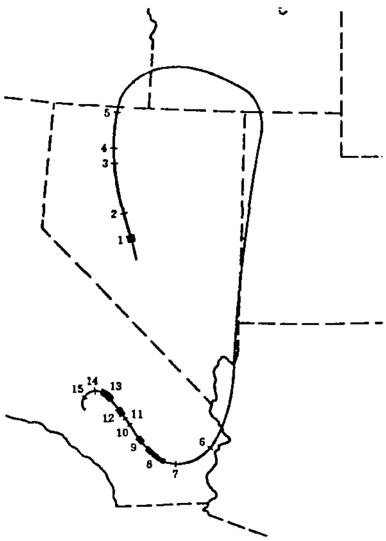
(b) 19 March 1965

Figure 5.- Examples of the turbulence data reported by the XB-70 aircraft in the stratosphere (after Ehernberger, ref. 1).



Encounter number	Greenwich time	$\Delta a_{n_{max}}$, g units	Distance in turbulence, n. mi. (km)	h_p, ft (m)
1	1456Z	0.15	2.2 (4.1)	65.0×10^3 (19.8×10^3)

(c) 27 July 1965



Encounter number	Greenwich time	$\Delta a_{n_{max}}$, g units	Distance in turbulence, n. mi. (km)	h_p, ft (m)
1	1638Z	0.15	8.5 (15.7)	42.0×10^3 (12.8×10^3)
2	1641Z	.25	1.1 (2.0)	48.0 (14.6)
3	1645Z	.10	.7 (1.3)	55.7 (17.0)
4	1645Z	.20	.7 (1.3)	56.0 (17.1)
5	1648Z	.10	2.9 (5.4)	58.6 (17.9)
6	1723Z	.10	1.6 (3.0)	58.1 (17.7)
7	1724Z	.25	1.2 (2.2)	58.7 (17.9)
8	1727Z	.20	34.7 (64.3)	57.4 (17.5)
9	1729Z	.20	9.3 (17.2)	56.0 (17.1)
10	1730Z	.10	1.8 (3.3)	56.4 (17.2)
11	1731Z	.15	4.9 (9.1)	55.9 (17.0)
12	1731Z	.15	5.0 (9.3)	54.0 (16.5)
13	1733Z	.35	24.5 (45.4)	52.0 (15.8)
14	1734Z	.25	3.6 (6.7)	50.2 (15.3)
15	1735Z	.10	4.5 (8.3)	42.4 (12.9)

(d) 16 October 1965

Figure 5.- Concluded.

Meteorological.- In the western United States, numerous mountain ranges have crests near the 3-km level (ref. 21). Thus, the 700-mb level is highly representative of meteorological conditions near mountain-top level. A 700-mb chart was plotted for each XB-70 flight. In addition, wind, temperature, and height data for 500 mb were plotted on each 700-mb chart. All meteorological information was obtained from the 1200 or 0000 GMT checked radiosonde and rawinsonde data available on microfilm from the National Weather Records Center. The wind component perpendicular to the mountains at mountain-top level, pertinent features of the wind profile in the low and middle troposphere, and the 700- to 500-mb lapse rate of temperature were data readily available from the analysis of these charts. This information helped to determine if atmospheric conditions in the troposphere might be favorable for the development of mountain waves.

Meteorological data from the rawinsonde stations shown in figure 6 were used for the analytical and statistical analyses in this research.

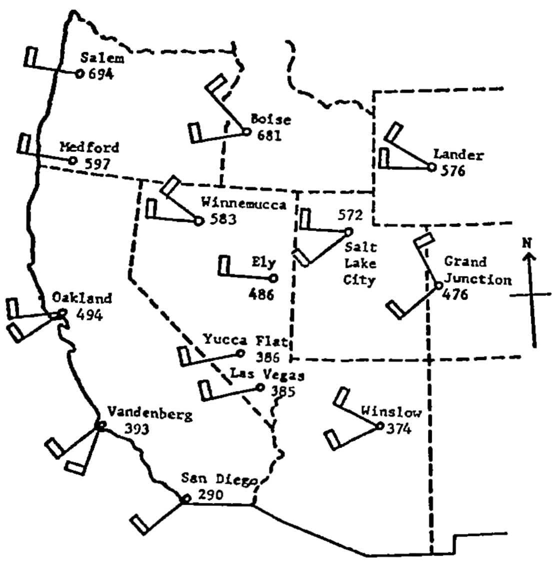


Figure 6.- Rawinsonde stations in the western United States and the prevailing wind directions favorable for the development of extensive mountain-wave activity in the surrounding areas (wind directions taken from Harrison and Sowa, ref. 21).

Analytical Results

The normal wind component at 700 mb.- Harrison and Sowa (ref. 21) studied areas of mountain waves along numerous commercial air routes across the western United States. Their results showed the location and relative intensity of mountain waves. The prevailing wind directions capable of causing extensive, rather than isolated, mountain waves in the areas surrounding the rawinsonde stations in the western United States are shown in figure 6. In general, a southwest through west wind is required for extensive development of mountain waves. At some stations, extensive development may occur when the wind is from either direction shown in the figure.

Harrison (ref. 22) developed a nomogram for forecasting the mountain wave near Denver, Colorado. A minimum wind speed of 10 m sec^{-1} was necessary between 3 and 5.5 km to produce mountain wave activity. Further application of the nomogram (ref. 23) substantiated the initial results.

A normal wind component of 10 m sec^{-1} at mountain-top level was tested in this study as the minimum speed required to initiate mountain waves leading to stratospheric turbulence. This speed is consistent with the nomogram developed by Harrison (refs. 22 and 23) and the theoretical considerations presented by Gazzola (ref. 11).

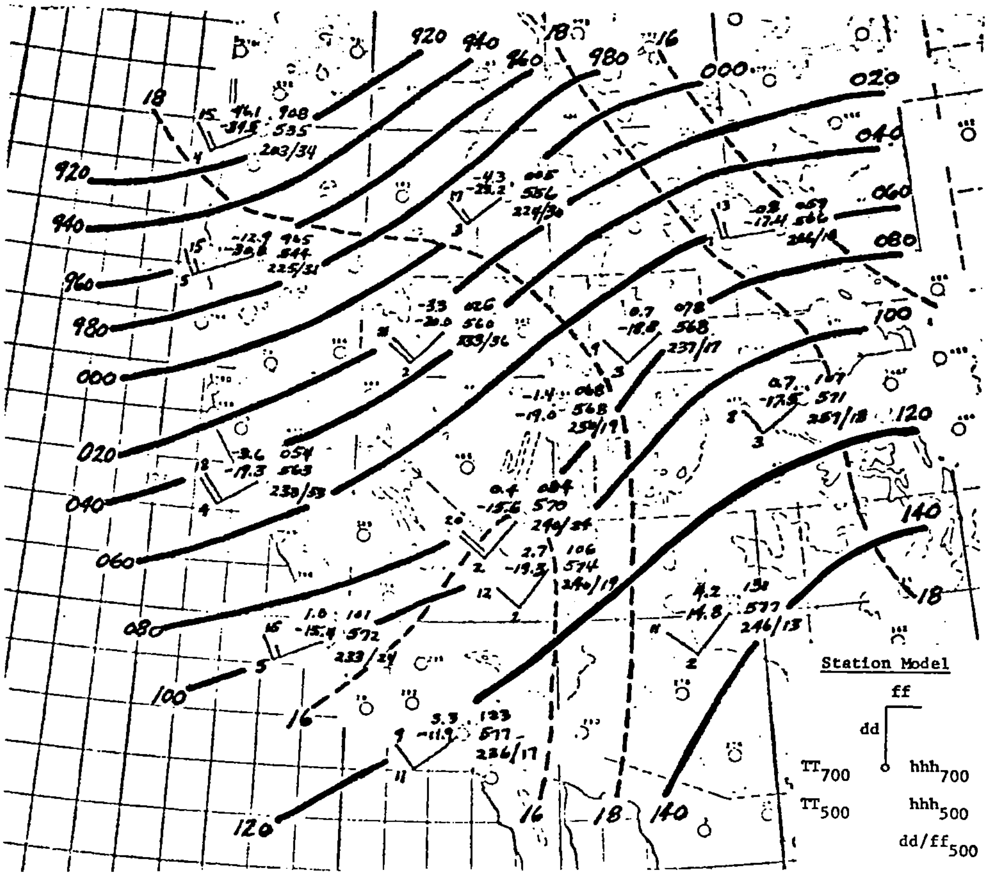
The wind component normal to the mountain crests was determined from the 700-mb data over the western United States. This wind component and the basic synoptic features of the 700-mb level exhibited an excellent correlation with the occurrence of turbulence in the stratosphere. In eight of nine cases in which the wind component normal to the mountains was greater than 10 m sec^{-1}, the XB-70 encountered turbulence in the stratosphere. In 18 of 22 cases, the normal wind component at mountain-top level was less than 10 m sec^{-1} and no stratospheric turbulence was encountered. In the five remaining cases, a closed low at 700 mb was observed within 5 deg of latitude of the XB-70 flight path, and stratospheric turbulence was reported in all five cases. These results are summarized in table II. Other data in the table will be discussed later.

TABLE II

SYNOPTIC FEATURES AT 700 MB AND THE AVERAGE VALUES ALONG THE FLIGHT TRACK FOR THE 700- TO 500-MB LAPSE RATE OF TEMPERATURE RELATED TO THE CONDITIONS IN THE STRATOSPHERE REPORTED BY THE XB-70

	Turbulence		No turbulence	
Synoptic features at 700 mb	Total cases	Individual lapse rate values 700-500 mb, °C	Total cases	Individual lapse rate values 700-500 mb, °C
Wind component normal to mountain crests ≥ 10 m sec^{-1}	8	16,16,16,16, 18,18,18,20	1	20
Wind component normal to mountain crests < 10 m sec^{-1}	4	12,14,18,20	18	14,14,16,16,16, 16,16,16,18,18, 18,18,20,20,20, 20,22
Closed low within 5 deg latitude of aircraft track	5	14,14,16,18, 20	0	

Examples of the various mid-troposphere conditions are shown in figures 7 through 10. The turbulence encountered in the stratosphere by the XB-70 in each of these cases is shown in figure 5. In figure 7, the wind component perpendicular to the mountain tops exceeds 10 m sec^{-1} in northern California, Oregon, Idaho, Nevada, and Wyoming, and over these areas 10 separate instances of turbulence were reported. In figure 8, the normal wind component at mountain-top level was less than 10 m sec^{-1}, yet the XB-70 encountered 15 turbulent locations over southern California and southern Nevada. A high



-------- 700-500 mb lapse rate of temperature, °C

——— Height above mean sea level of 700 mb surface in tens of meters, e.g., 040 represents 3040 m

dd - Direction from which wind is blowing measured clockwise from true north

ff - Wind speed in m sec^{-1} -- a full barb represents 10 m sec^{-1} and a half barb 5 m sec^{-1}

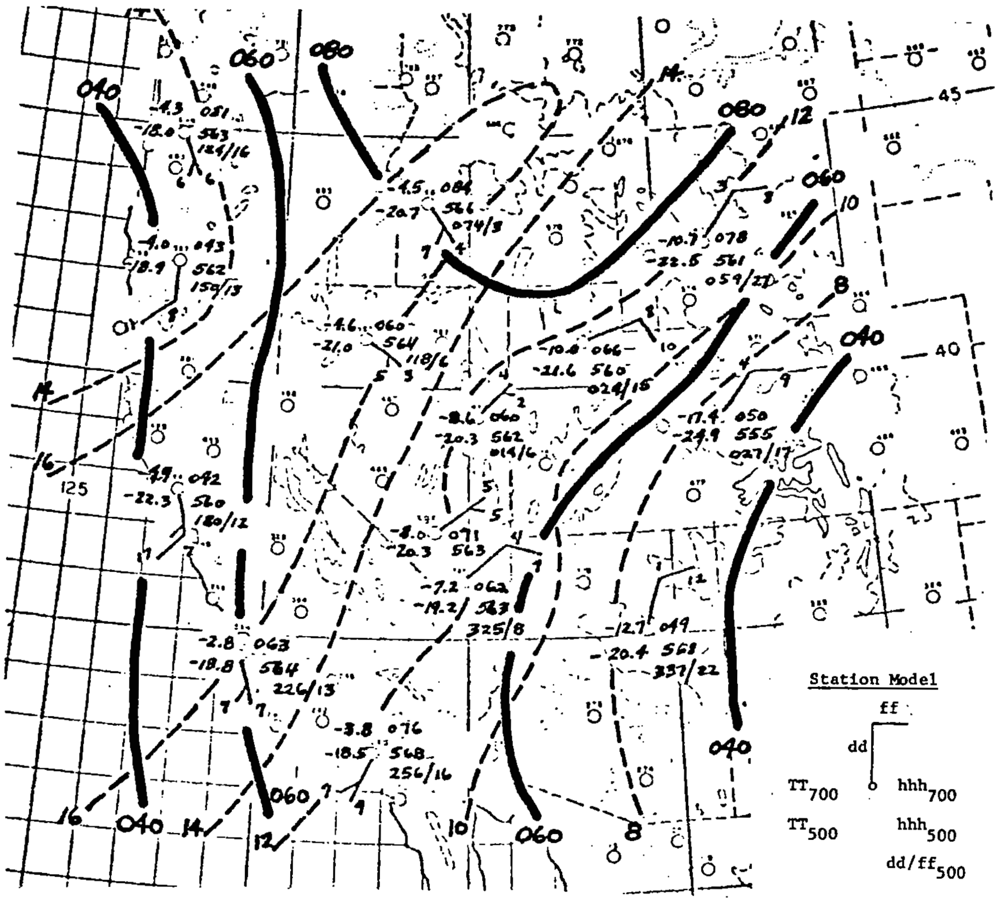
hhh - Height in tens of meters above mean sea level

dd/ff - Wind direction and speed

TT - Temperature, °C

Subscripts - Mb surface at which the variable was measured

Figure 7.- Mid-tropospheric conditions for 1200 GMT on 19 March 1966.



-------- 700-500 mb lapse rate of temperature, °C

▬▬▬ Height above mean sea level of 700 mb surface in tens of meters, e.g., 040 represents 3040 m

dd - Direction from which wind is blowing measured clockwise from true north

ff - Wind speed in m sec^{-1} -- a full barb represents 10 m sec^{-1} and a half barb 5 m sec^{-1}

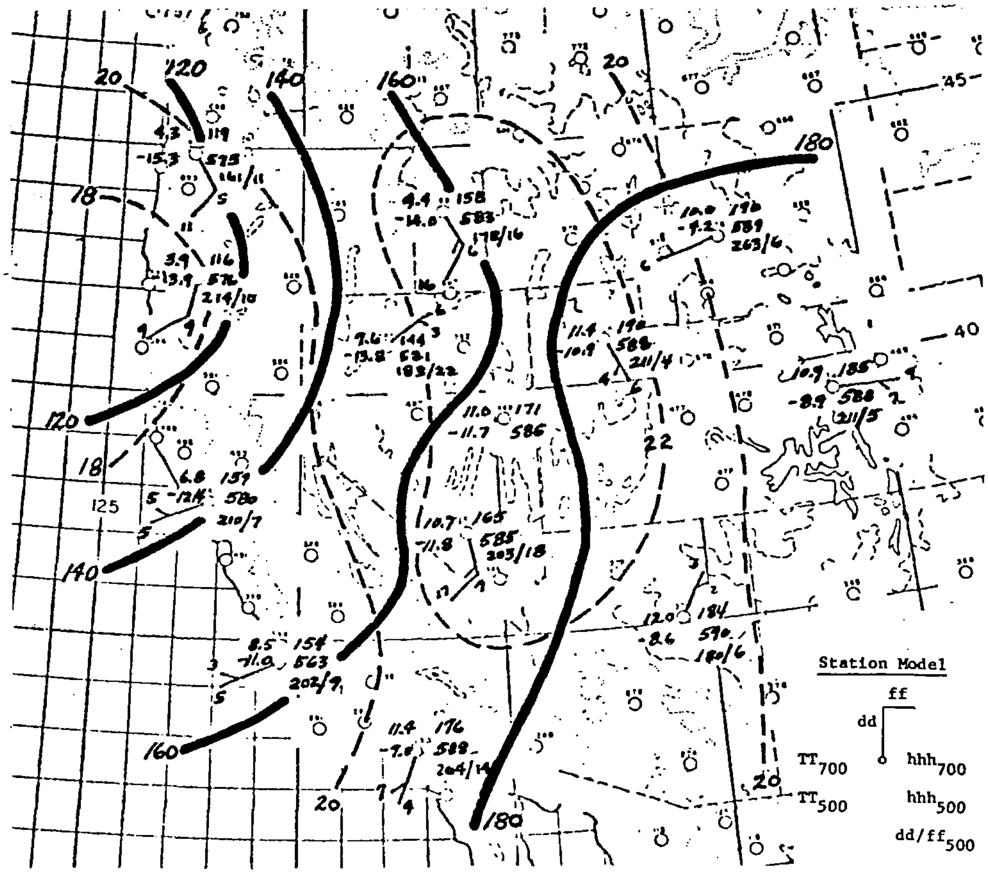
hhh - Height in tens of meters above mean sea level

dd/ff - Wind direction and speed

TT - Temperature, °C

Subscripts - Mb surface at which the variable was measured

Figure 8.- Mid-tropospheric conditions for 1200 GMT on 4 March 1965.



---------- 700-500 mb lapse rate of temperature, °C

━━━━━━━━ Height above mean sea level of 700 mb surface in tens of meters, e.g., 040 represents 3040 m

dd - Direction from which wind is blowing measured clockwise from true north

ff - Wind speed in m sec^{-1} -- a full barb represents 10 m sec^{-1} and a half barb 5 m sec^{-1}

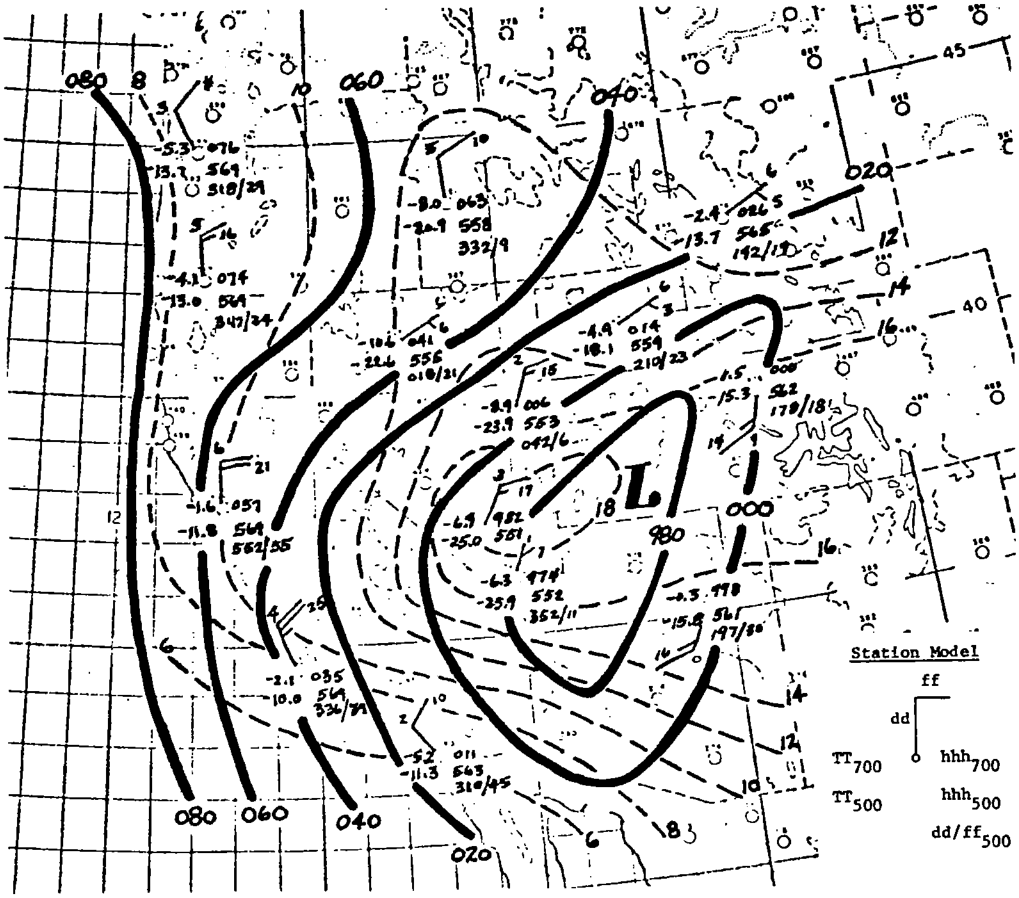
hhh - Height in tens of meters above mean sea level

dd/ff - Wind direction and speed

TT - Temperature, °C

Subscripts - Mb surface at which the variable was measured

Figure 9.- Mid-tropospheric conditions for 1200 GMT on 27 July 1965.



-------- 700-500 mb lapse rate of temperature, °C

———— Height above mean sea level of 700 mb surface in tens of meters, e.g., 040 represents 3040 m

dd - Direction from which wind is blowing measured clockwise from true north

ff - Wind speed in a m sec^{-1} -- a full barb represents 10 m sec^{-1} and a half barb 5 m sec^{-1}

hhh - Height in tens of meters above mean sea level

dd/ff - Wind direction and speed

TT - Temperature, °C

Subscripts - Mb surface at which the variable was measured

Figure 10.- Mid-tropospheric conditions for 1200 GMT on 16 October 1965.

frequency of turbulence, such as this, with winds at mountain-top level unfavorable for the development of mountain waves occurred in just four of the 17 cases classified as turbulence cases.

The 700-mb conditions when only one isolated instance of stratospheric turbulence occurred over northern Utah are shown in figure 9. In this no-turbulence case, the normal wind component at mountain-top level was everywhere less than 10 m sec^{-1}. In figure 10, a closed low was centered over southern Utah, and 15 instances of turbulence were reported over southern California and Nevada. In all of the closed-low cases, no specific quadrant was favored for the location of the stratospheric turbulence.

The 700- to 500-mb lapse rate of temperature.- For each rawinsonde station, the 700- to 500-mb lapse rate of temperature was determined, and these data were analyzed in 2 °C intervals. An average value of this lapse rate was determined for each flight track of the XB-70. The average values for this lapse rate of temperature in °C are shown in table II for the turbulence and no-turbulence cases and separated according to the 700-mb features.

As an individual parameter, the lapse rate of temperature did not successfully differentiate between cases of turbulence and no turbulence. The lapse rate of temperature in combination with the 700-mb wind data did not improve the results obtained by the normal wind component alone. However, the data seem to suggest that the more stable lapse rates of temperature tend to be associated with the occurrence of stratospheric turbulence. Perhaps if the temperature data were concurrent with the turbulence reports, the results would be improved.

The gradient and advection of stability.- Large horizontal gradients of the 700- to 500-mb lapse rate of temperature were observed to be associated with turbulence cases. A gradient of at least 3 to 4 °C per 300 n mi along the flight path was observed in these cases, and gradients as high as 5 to 8 °C per 300 n mi occurred also. In the cases of no turbulence in the stratosphere, the gradients ranged from 1 to 4 °C per 300 n mi.

In most cases, more stable, rather than unstable, air was advected at tropospheric levels into the areas where turbulence occurred. However, several cases of neutral or less stable advection did not allow any firm conclusions to be made.

Some of the features of the 700- to 500-mb lapse rate of temperature are shown in figures 7 through 10. In figure 7, most of the stratospheric turbulence occurred over the relatively stable area of western Nevada and also over Idaho and Wyoming. The horizontal gradient over these areas is 3 °C per 300 n mi. In figure 8, the turbulence occurred in the most stable area that has an associated gradient of 5 °C per 300 n mi in south-central California and southern Nevada. In figure 10, the horizontal gradient of the 700- to 500-mb lapse rate of temperature (stability) is 7 °C per 300 n mi along the flight path over southern California where the turbulence was reported.

The horizontal gradients of the stability estimated from figure 9 range from 1 to 4 °C per 300 n mi for the flight track over western Nevada, southeastern Oregon, southern Idaho, western Utah, northwestern Arizona and southern California. During this flight, which was classified as a no-turbulence case, the XB-70 reported only one instance of turbulence.

The results of this stability analysis are shown in table III and indicate that a horizontal gradient of at least 5 °C per 300 n mi for the 700- to 500-mb lapse rate of temperature is associated with turbulence in the stratosphere. A gradient of 2 °C or less per 300 n mi is related to non-turbulent conditions in the stratosphere. For gradients of 3 to 4 °C per 300 n mi, the results are inconclusive since these values are associated with cases of turbulence as well as no-turbulence.

TABLE III

THE HORIZONTAL GRADIENT OF THE 700- TO 500-MB LAPSE RATE OF TEMPERATURE RELATED TO CONDITIONS AT 700 MB AND TURBULENCE REPORTED IN THE STRATOSPHERE BY THE XB-70

Synoptic features at 700 mb	Turbulence	Cases	Horizontal gradient of the 700- to 500-mb lapse rate of temperature per 300 n mi		
			≤ 2 °C	3-4 °C	≥ 5 °C
Wind component normal to mountain crests ≥ 10 m sec^{-1}	Yes	8	0	3	5
	No	1	0	1	0
Wind component normal to mountain crests < 10 m sec^{-1}	Yes	4	1	2	1
	No	18	9	9	0
Closed low within 5 deg latitude of aircraft track	Yes	5	0	2	3
	No	0	0	0	0

Scorer parameter and Richardson number profiles.- The profiles of L^2 and Ri for each XB-70 flight were computed from the reported rawinsonde data from the stations near the flight track of the aircraft. The computer program which was mentioned previously was utilized for the computations.

For each of the 36 cases, the L^2 and Ri profiles were averaged for two to five stations. The number of stations depended upon the location and length of the flight track. Only in isolated instances was it necessary to omit a station from the calculations because of insufficient data.

The average profiles of L^2 for turbulence cases which occurred with a normal wind component of at least 10 m sec^{-1} at 700 mb showed strong similarities to the profiles for the atmospheric models shown in figure 1 and considered representative of mountain-wave conditions. Figure 11 shows the average profiles of L^2 for eight cases of this type. In general, these profiles match the model profiles for mountain-wave conditions in shape and magnitude, and they have a minimum value at 400 or 300 mb. Several of these cases show about an order-of-magnitude decrease in the value of L^2 from the mountain-top level to the minimum value in the middle or upper troposphere.

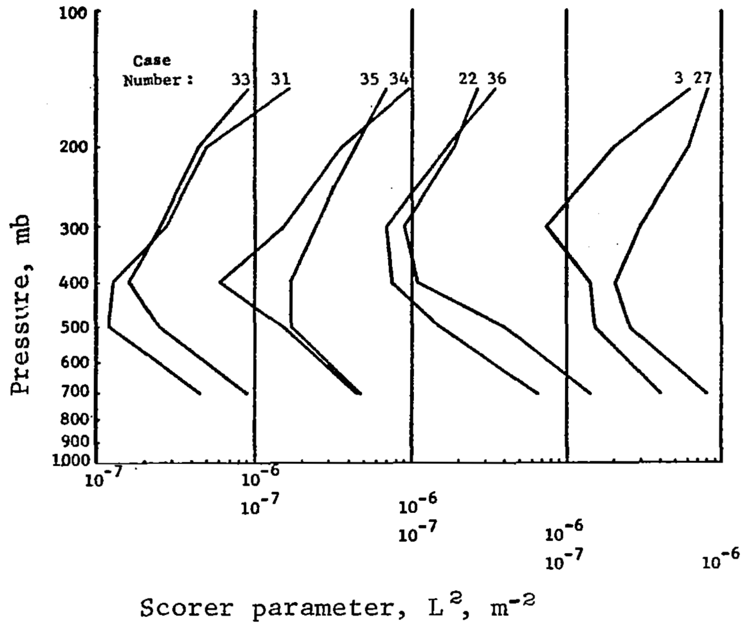


Figure 11.- Eight average profiles of L^2 when the normal wind component at the mountain crests was at least 10 m sec^{-1} and conditions reported in the stratosphere by the XB-70 were classified as turbulence cases.

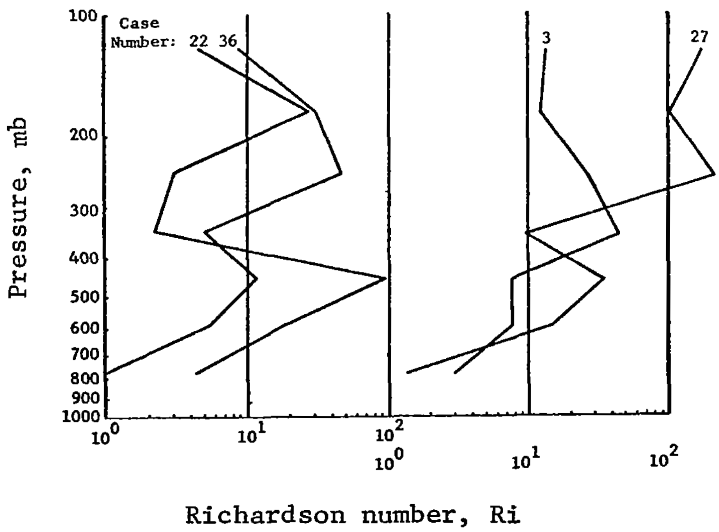


Figure 12.- Average profiles of Ri which correspond to the first four profiles of L^2 in figure 11.

The average profiles of Ri for these eight turbulence cases are shown in figures 12 and 13. In general, the profiles display an increase in the value of Ri from the mountain tops to a level in the middle troposphere, a decrease into the upper troposphere, and above this a continuous increase into the stratosphere. These average profiles of Ri show reasonable similarities to the model profiles of Ri for mountain-wave conditions (see figure 3). In a few instances, for these and other cases, it was necessary to delete an exceptionally large value of Ri from the averaging process because of the excessive distortion of the profile. But, never was more than one value for a specific level deleted from the calculations.

Examples of average profiles of L^2 are shown in figure 14 for cases in which conditions in the troposphere were not favorable for the development of waves. The differences in the shape and magnitude of these profiles in comparison to the mountain-wave cases shown in figure 11 are obvious. Since some of these profiles represent turbulence cases, it can be concluded that the turbulence must have been caused by some event other than mountain waves. This conclusion is supported further by the accompanying profiles of Ri shown in figure 15. The shape and magnitude of these Ri profiles differ considerably from those of the mountain-wave cases shown in figures 12 and 13.

Some of the average profiles of L^2 for the no-turbulence cases were similar in shape and magnitude to those

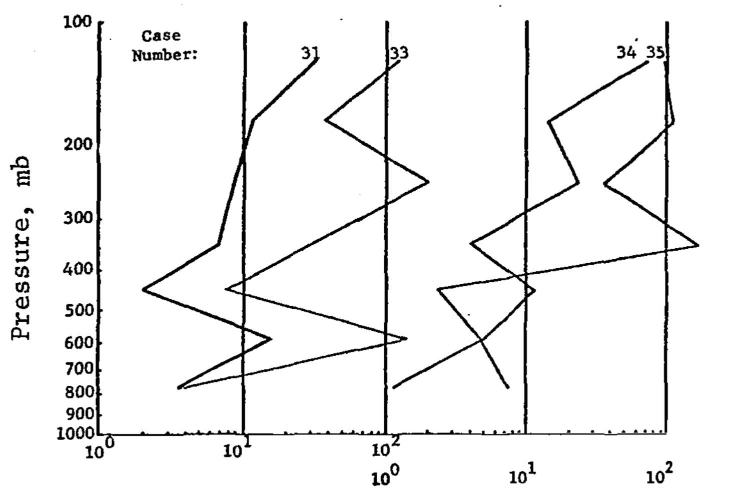


Figure 13.- Average profiles of Ri which correspond to the last four profiles of L^2 in figure 11.

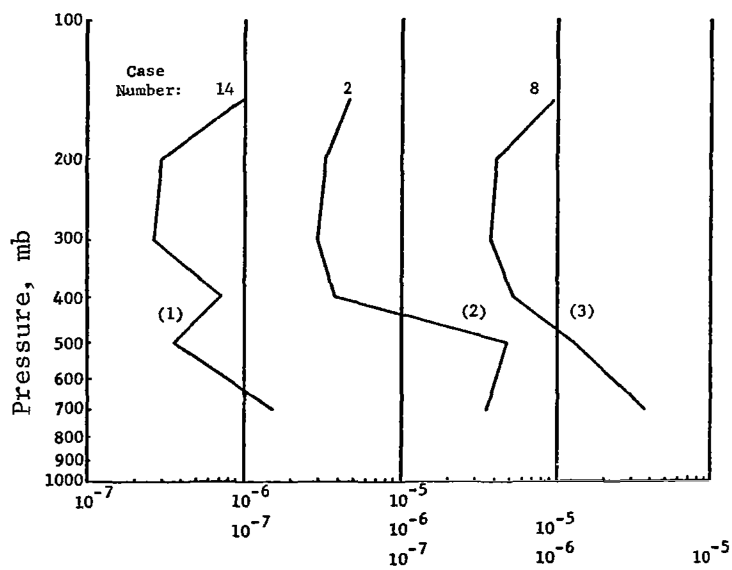


Figure 14.- Average profiles of L^2 for specific conditions at 700 mb and in the stratosphere: (1) A closed low at 700 mb within 5 deg of latitude of the flight track of the XB-70 and classified as a turbulence case; (2) the wind component normal to the mountain tops less than 10 m sec^{-1} and classified as a turbulence case; (3) the wind component normal to the mountain tops less than 10 m sec^{-1} and classified as a no-turbulence case.

associated with mountain waves. Investigation of these cases showed wind profiles with strong north or south wind speeds but without the normal wind component of at least 10 m sec^{-1} at 700 mb. These particular cases must be considered potential mountain-wave cases since only a change in the wind direction would be required to meet the minimum wind criterior of 10 m sec^{-1} normal to the mc ntain crests.

The most characteristic feature of the model profiles of L^2 and Ri is the minimum point in the middle or upper troposphere. From the model profiles of L^2 and Ri, a minimum value in the middle or upper troposphere was selected that appeared to differentiate between conditions with and without mountain waves. The selected value of L^2 was 3 x $10^{-7} m^{-2}$, and that for Ri was 10.

For the analysis of the minimum values of the average profiles of L^2 and Ri in the middle or upper troposphere, the 36 cases in this study were grouped according to slightly different criteria than used previously. All the cases with an observed or potential wind component of 10 m sec^{-1} or more normal to the mountain tops were grouped together along with all the closed-low cases. The remaining cases with observed wind components less than 10 m sec^{-1} normal to the mountain crests comprised the second group of cases. The results of this analysis are shown in figures 16 and 17.

For the strong wind and closed-low cases, the minimum

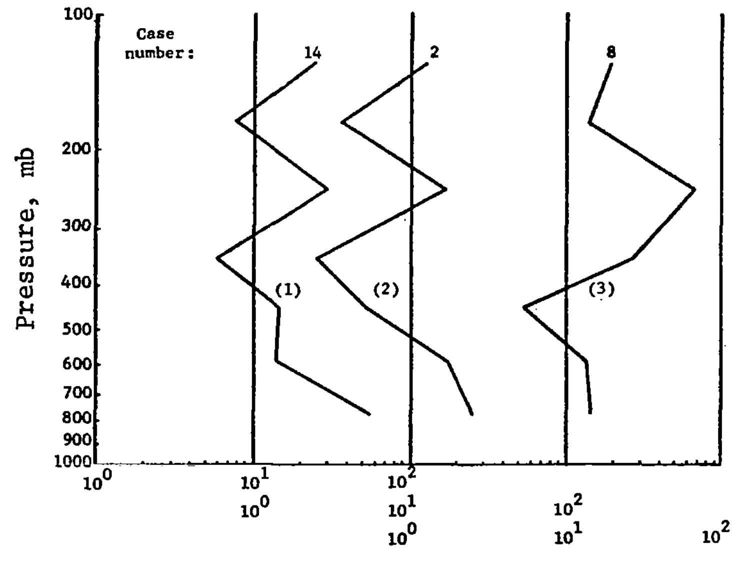


Figure 15.- The average profiles of Ri for the cases and conditions specified for the corresponding profiles of L^2 in figure 14.

profile value of $3 \times 10^{-7} m^{-2}$ for L^2 was exceeded only twice, and both of these cases involved a closed low at mountain-top level. All but two of these cases had a minimum value of Ri less than 10. For the light-wind cases, the minimum profile value of L^2 was greater than $3 \times 10^{-7} m^{-2}$ in 13 of 18 cases. For the five other cases, the minimum value of L^2 was less than $3 \times 10^{-7} m^{-2}$, but turbulence was reported in only two of these cases. The minimum value of Ri was greater than 10 in six cases in which no turbulence occurred and the minimum value of L^2 exceeded $3 \times 10^{-7} m^{-2}$. For the 12 remaining cases, the minimum profile value of Ri was less than 10.

The magnitude of the minimum profile value for L^2 differentiates between the actual or potential turbulence and no-turbulence cases extremely well, whereas Ri does not. The results are summarized in table IV. For those cases with an actual or potential wind component normal to the mountain tops and greater than or equal to 10 m sec^{-1}, but excluding closed-low cases, the minimum magnitude of L^2 was less than $3 \times 10^{-7} m^{-2}$ in 13 out of 13 cases. Turbulence was reported in the stratosphere in 12 of these 13 cases. For the four cases of light winds and reported stratospheric turbulence, $3 \times 10^{-7} m^{-2}$ was exceeded twice. In the light wind cases with no turbulence, $3 \times 10^{-7} m^{-2}$ was exceeded in 11 of 14 cases. The closed-low cases

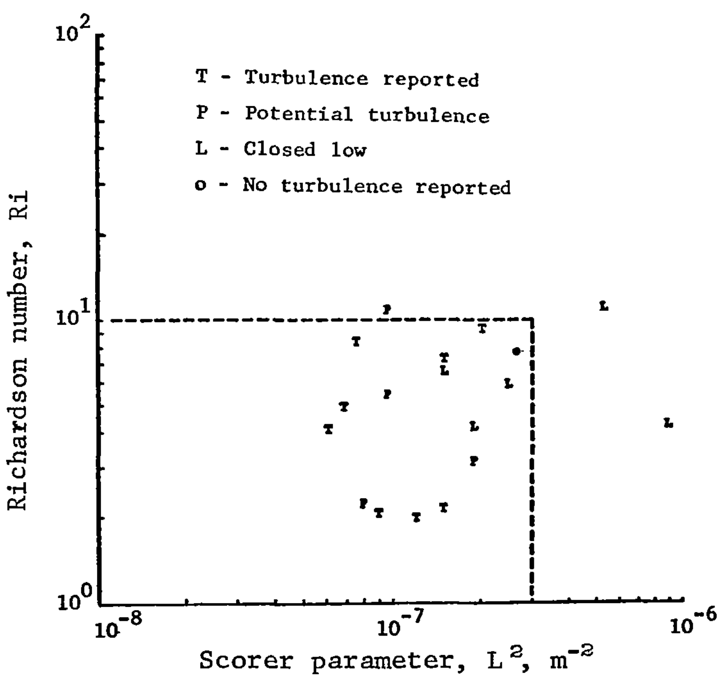


Figure 16.- The minimum value in the middle or upper troposphere of the average profiles of L^2 and Ri for those cases with actual and potential wind components of at least 10 m sec^{-1} normal to the mountain tops and for all closed-low areas within 5 deg of latitude of the flight track of the XB-70. (NOTE: The minimum wind criterion was not considered in the closed-low cases.)

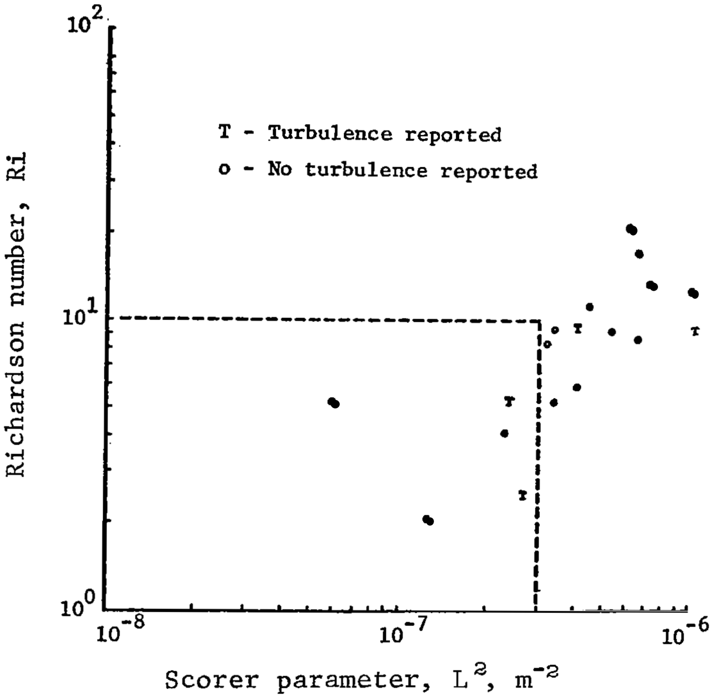


Figure 17.- The minimum value in the middle or upper troposphere of the average profiles of L^2 and Ri for those cases with wind components less than 10 m sec^{-1} normal to the mountain tops. (NOTE: Closed-low cases are not considered in this figure.)

were added to the aforementioned results according to wind velocities only. The results are indicated by the figures in parentheses in table IV.

The minimum profile value of Ri did not produce any conclusive results as far as separating cases of turbulence and no turbulence. Under light wind conditions, no turbulence occurred when the minimum profile value of Ri exceeded 10. In virtually all of the turbulence cases, the minimum profile value of Ri was less than 10; however, a similar result occurred for more than half the cases in which there was no turbulence in the stratosphere. Thus, the Ri value, by itself, does not successfully differentiate between cases of turbulence and no turbulence. Finally, Ri in combination with L^2 does not improve the results obtained by using L^2 alone.

TABLE IV

TURBULENCE OR POTENTIAL TURBULENCE CONDITIONS IN THE STRATOSPHERE RELATED TO THE MINIMUM VALUE OF THE AVERAGE PROFILE OF L^2

	Turbulence or potential turbulence		No turbulence	
Actual or potential wind component normal to the mountain crests	L^2 minimum profile value, m^{-2}		L^2 minimum profile value, m^{-2}	
	$< 3 \times 10^{-7}$	$> 3 \times 10^{-7}$	$< 3 \times 10^{-7}$	$> 3 \times 10^{-7}$
≥ 10 m sec^{-1}	12 (2)*	0 (1)	1	0
< 10 m sec^{-1}	2 (1)	2 (1)	3	11

* Closed-low cases in parentheses.

Vertical wind shear.- As discussed by Booker and Bretherton (ref. 19) and others, the vertical wind shear plays an important role in the vertical propagation of wave motions and energy transfer and, therefore, it is an important consideration in the investigation of mountain waves and stratospheric turbulence. The terms in equation (2) were computed separately, and the ratio of the first to the second term was calculated in order to determine the importance of the shear term to the value of L^2. The results of this analysis are shown in table V. Only the data from the stations used to determine the average profiles of L^2 and Ri are included. For example, for the eight turbulence cases with the minimum wind component of 10 m sec^{-1} normal to the mountain tops, 27 stations were used to evaluate the average profiles of L^2. Of the 27 stations, four had no levels with a ratio of the first to the second term of equation (2) which was 1:1 or less. Eight stations had one level with the 1:1 or less ratio. Seven stations had two such levels, six stations had three levels, and two stations exhibited four levels with a 1:1 or less ratio of terms.

TABLE V

THE RATIO OF THE FIRST TO THE SECOND TERM OF THE SCORER PARAMETER VERSUS THE REPORTED CONDITIONS IN THE STRATOSPHERE AND TROPOSPHERE

			Levels with ratio* ≤ 1:1				
			0	1	2	3	4
Synoptic features at 700 mb	Conditions in stratosphere	Total cases	Number of stations with ratio of terms 1:1 or less				
Wind component normal to mountain crests ≥ 10 m sec^{-1}	Turbulence	8	4	8	7	6	2
	No turbulence	1	1	2	0	0	0
Potential wind component normal to mountain crests ≥ 10 m sec^{-1}	Potential turbulence	4	3	2	3	2	1
Wind component normal to mountain crests < 10 m sec^{-1}	Turbulence	4	9	6	1	0	0
	No turbulence	14	31	12	4	2	0
Closed low within 5 deg latitude of aircraft track	Turbulence	5	5	5	7	4	1
	No turbulence	0	0	0	0	0	0

* Ratio = $\frac{g\beta}{U^2} \Big/ \frac{U''}{U}$

In general, the magnitude of the wind shear term is important to the value of L^2 for mountain wave and potential mountain wave cases as well as for the closed-low cases. With the minimum wind component of 10 m sec^{-1} normal to the mountain tops, 85 percent of the stations had at least one level where the ratio of the first to the second term of equation (2) was 1:1 or less. For 56 percent of the stations, two or more levels had the 1:1 ratio or less. For the potential mountain-wave cases, 73 percent of the stations showed at least one level with a 1:1 ratio or less. In the closed-low cases, about 78 percent of the stations had at least one level with a 1:1 ratio of terms, while 60 percent of the stations had two or more levels.

For the light-wind cases, the shear term displayed a marked decrease in its importance to the value of L^2. In four cases when stratospheric turbulence was reported, nearly 56 percent of the stations showed no levels with a 1:1 ratio of terms. In the no-turbulence cases, about 63 percent of the stations had no levels with a 1:1 ratio of terms.

The 1:1 ratio or less of the terms in equation (2) occurred most frequently near the tropopause. However, the results in table V suggest that the vertical wind shear plays an important role in layers other than just the surface boundary layer and near the jet stream as stated by Foltz (ref. 6). The effect of this shear may be extremely important in the vertical propagation and transfer of energy when mountain waves develop (refs. 19 and 20).

The magnitude of the vertical wind shear between individual levels compared favorably with the shears in the model profiles for strong winds. Vertical shears of about 8 m sec^{-1} per 2000 m from the mountain tops to the tropopause represent a reasonable minimum value of these shears. The fact that one no-turbulence case occurred with the necessary normal wind component at 700 mb can be attributed to very small vertical wind shears.

Finally, for all the mountain-wave cases, there was little change in the wind direction in the troposphere between 3 and 10 km.

Evidence of the presence of mountain waves from the variation in ascent rate of rawinsonde balloons.- Corby (ref. 24) has shown that rawinsonde balloons experience variations in ascent rate when they ascend through wave motions in the atmosphere. Booker (ref. 25) found also that a floating balloon experienced sizable up and down motions in the presence of lee waves over central Pennsylvania. Lee waves have been observed on many occasions in the mountainous regions of the western United States, and hence it is logical to expect that these waves would be manifested as changes in the ascent rate of rawinsonde balloons, particularly when the waves are well developed.

Vertical profiles of the ascent rate of rawinsonde balloons at a number of locations were computed on days corresponding to the XB-70 flights. The ascent rates were computed from position data at 2-min intervals. Examples of ascent rate profiles are shown in figure 18. The first two profiles, computed from data at 00Z, 21 April 1965, represent conditions favorable for the development of mountain waves, while the two profiles in this figure, computed from data at 12Z, 28 April 1965, represent conditions unfavorable for the development of mountain waves. On 21 April 1965, large and cyclic variations in the ascent rates of the balloons were observed, while on 28 April 1965 variations in the ascent rates were much smaller and non-cyclic. Profiles displaying these characteristics were found to be typical for each case.

Ascent rate profiles were computed on each date corresponding to the flight of the XB-70, and the data were grouped as discussed previously, i.e., based on the magnitude of the wind component normal to the mountains and the presence of a closed low at mountain-top level. Each profile was analyzed to determine if large amplitude and cyclic variations were present and, if so,

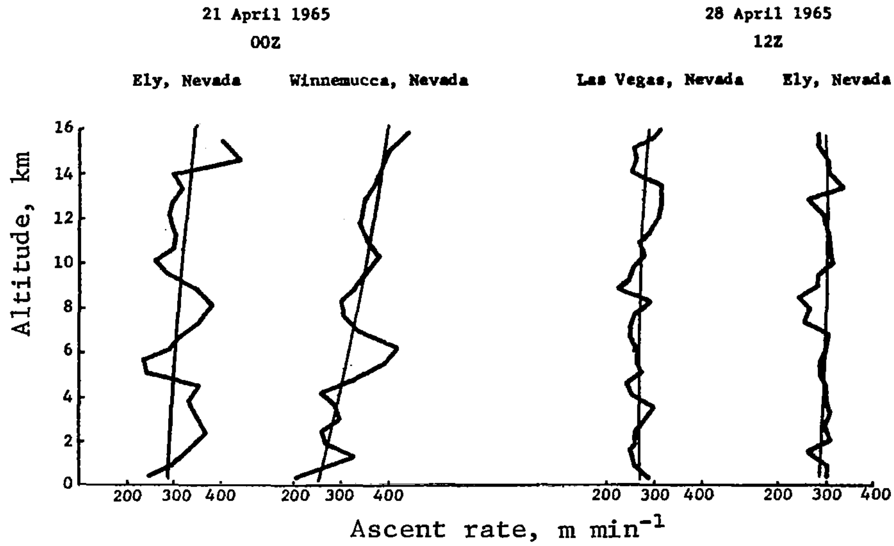


Figure 18.- Examples of the ascent rate of rawinsonde balloons indicative of the presence of mountain waves (21 April 1965) and the absence of such waves (28 April 1965). The smooth lines represent average ascent rate profiles drawn by eye.

this was considered as indicative of the presence of mountain waves. The results are shown in table VI. The column headed "Case no." in table VI has no meaning other than for identification of the flights. A "Yes" is indicated when the ascent rate curves contained large amplitude and cyclic variations similar to those shown in the first two profiles in figure 18, and a "No" indicates the absence of these characteristics with the profiles being similar to the last two in figure 18. It is clear from table VI(a) that when the wind component normal to the mountain ranges is ≥ 10 m sec^{-1}, mountain waves are indicated in most of the ascent rate profiles. As pointed out earlier, turbulence was generally associated with these conditions. Of the profiles examined in this category, only 18 percent did not show cyclic variations with large amplitude. For the group of data in table VI(d) when the wind component normal to the mountains was < 10 m sec^{-1}, only 6 percent of the profiles examined had cyclic variations with large amplitudes. These two categories differentiate quite well between turbulence and no-turbulence conditions. In the case when the wind component normal to the mountains was < 10 m sec^{-1}, and the aircraft encountered turbulence, only 28 percent of the profiles indicated the presence of mountain waves. In the closed-low cases shown in table VI(e), only 19 percent of the profiles examined indicated the presence of mountain waves.

Smooth ascent rate profiles were drawn by eye for each rawinsonde sounding considered in table VI. Examples are shown in figure 18. The magnitude of the differences between the average and measured profiles at the maximum and minimum points on the measured profiles were computed. The maximum difference usually was considerably larger than all other values, and was eliminated from further consideration on the basis that it probably resulted from

TABLE VI

INDICATION OF THE PRESENCE OR ABSENCE OF MOUNTAIN WAVES FROM THE VARIATION IN ASCENT RATE OF THE RAWINSONDE BALLOON

Case no.	Date	Time (Z)	Location *						
			1	2	3	4	5	6	7
(a) Turbulence - wind component normal to mountains ≥ 10 m sec^{-1}									
3	20 Apr 65	12	Yes	Yes	Yes				
	21 Apr 65	00	Yes	Yes	Yes				
22	2 Dec 65	00		No		Yes			
	2 Dec 65	12	Yes	Yes		No	Yes		
	3 Dec 65	00	Yes	Yes		No	Yes		
27	3 Jan 66	12		Yes		Yes	Yes		
	4 Jan 66	00				Yes	Yes		
31	9 Feb 66	12			Yes				
	10 Feb 66	00	No		Yes				
33	10 Mar 66	12		Yes		Yes	No		
	10 Mar 66	18					Yes		
	11 Mar 66	00					Yes		
34	15 Mar 66	12		No		Yes			
	16 Mar 66	00		Yes					
35	17 Mar 66	12	Yes	Yes		Yes			
	18 Mar 66	00	No						
36	19 Mar 66	12		Yes					
	20 Mar 66	00		Yes		Yes			
(b) No turbulence - wind component normal to mountains ≥ 10 m sec^{-1}									
7	1 July 65	12	Yes	No			Yes		
(c) Turbulence - wind component normal to mountains < 10 m sec^{-1}									
2	4 Mar 65	12	No		No			No	
	5 Mar 65	00	No		No			No	
11	22 Sept 65	12	Yes	Yes	Yes				
	23 Sept 65	00	No	No	Yes		No		
16	2 Nov 65								
	3 Nov 65	00		No			No		
29	12 Jan 66	12	Yes	No					No **
(d) No turbulence - wind component normal to mountains < 10 m sec^{-1}									
1	16 Feb 65								
4	28 Apr 65	12	No	No	No				
	29 Apr 65	00	No	Yes	No				
8	27 Jul 65	12	No						
12	30 Sept 65	00	No	No		Yes	No		
(d) Cont'd.									
13	14 Oct 65	12		No					
15	26 Oct 65								
17	4 Nov 65								
18	12 Nov 65	12	No						
19	18 Nov 65								
20	30 Nov 65	12	No	No	No		No		
	30 Nov 65	18					No		
21	1 Dec 65	00	No	No	No		No		
	1 Dec 65	12		No		No			
23	7 Dec 65	12		No					
	8 Dec 65	00		No					
24	10 Dec 65								
25	11 Dec 65	12	No						
	11 Dec 65	18	No						
	12 Dec 65	00	No						
26	21 Dec 65								
	22 Dec 65	00	No	No					
28	11 Jan 66								
30	15 Jan 66								
32	17 Feb 66	12	No						
	18 Feb 66	00	No	No		No			
(e) Closed low at mountain-top level									
5	7 May 65	12	Yes	No	No				
	8 May 65	00	Yes	No	No				
6	16 Jun 65	12	No		No		No		
	17 Jun 65	00			No				
9	18 Aug 65	12							No
	18 Aug 65	18							No
	19 Aug 65	00							No
10	17 Sept 65	12	No		No				
	18 Sept 65	00	No	No	Yes				
14	16 Oct 65	.12		No					

* Location code:

- 1 Las Vegas
- 2 Winnemucca
- 3 Ely
- 4 Boise
- 5 Salt Lake City
- 6 Vandenberg
- 7 Oakland
- ** Yucca

error in the measurements. The average magnitude and range of the near-maximum variations between the measured and average ascent rate profiles are given in table VII for each category in table VI. The average magnitude of the near-maximum variations in ascent rate for the "Yes" cases is about twice as large as that for the "No" cases. As shown in table VII, the range of these variations is larger for the "Yes" than for the "No" cases. Turbulence is associated with variations in ascent rate of the rawinsonde balloon (difference between average and measured ascent rate profiles) with near-maximum magnitudes between approximately 50-150 m min^{-1}. This is approximately one-fourth of the average ascent rate which is the value quoted in reference 26 when CAT was observed. The corresponding range for the non-turbulent areas is 30-70 m min^{-1}. In the overlap region between 50-70 m min^{-1}, turbulence cases are characterized by organized and cyclic variations about the average profile, while the non-turbulence cases are characterized by variations which, in most cases, are essentially random. The cyclic variations associated with the "Yes" cases were better organized and observed more often between 3 and 10 km than at higher altitudes, although they were observed on several profiles as high as 25 km.

From the data presented in table VII, there seems to be little doubt that turbulence is associated with conditions which lead to changes in the ascent rate of the rawinsonde balloons, and if these conditions are indicative of the presence of mountain waves, then there are strong indications that mountain waves in the western part of the United States were responsible primarily for the observed CAT on the days examined. Also, it appears possible to determine if CAT should be present on the basis of variations in the ascent rate of radiosonde balloons with a reliability of approximately 80 percent. It is possible that this percent could be further improved if the orientation and proximity of the ranges to the individual stations were considered.

TABLE VII

THE AVERAGE MAGNITUDE AND RANGE OF NEAR-MAXIMUM VARIATIONS IN ASCENT RATE OF RAWINSONDE BALLOONS FOR EACH CATEGORY GIVEN IN TABLE VI

Category from table VI	Average magnitude of near-maximum variations in ascent rate, m min^{-1}		Range of magnitude of near-maximum variations in ascent rate, m min^{-1}	
	Yes	No	Yes	No
(a)	74	34	40-180	30-60
(b)	80	40	*	*
(c)	72	43	50-120	30-70
(d)	85	40	70-100	20-80
(e)	97	47	80-110	20-70

* Insufficient data to establish range.

FORECASTING CAT IN THE STRATOSPHERE FROM CONDITIONS IN THE TROPOSPHERE

The results of this study have shown positive relationships between conditions in the troposphere favorable for mountain waves and the occurrence of turbulence in the stratosphere. From these results, objective techniques for forecasting CAT in the stratosphere are developed for use in the weather station and/or forecast center.

If any of the following conditions exist in the troposphere, turbulence should be forecast in the stratosphere: 1) The normal wind component at the crest of the mountains is greater than or equal to 10 m sec^{-1}, and the wind speed increases with height so as to produce wind shears of at least 8 m sec^{-1} per 2000 m (relatively little variation of the wind direction with height in the troposphere between 3 and 10 km was observed in the turbulence cases studied); 2) a closed low is present at mountain-top level within 5 deg of latitude of the flight track of the airplane; 3) the horizontal gradient of the 700- to 500-mb lapse rate of temperature is at least 5 °C per 300 n mi. From the data analyzed, forecasts based on the first two of the above criteria will verify about 80 percent of the time. The third criterion verifies 100 percent when such conditions are observed, but by itself it identifies only 50 percent of the turbulence cases. For the mountain-wave conditions, turbulence would be expected at distances of several tens of kilometers to the lee of the mountain ranges.

A computer can be utilized to compute the wind component normal to the mountain ranges and the average profile of L^2. If the normal wind component is at least 10 m sec^{-1} at mountain-top level, and the minimum value of the average profile of L^2 is less than $3 \times 10^{-7} m^{-2}$, forecast turbulence in the stratosphere. If the normal wind component is less than 10 m sec^{-1} at mountain-top level, and the minimum value of the average profile of L^2 is greater than $3 \times 10^{-7} m^{-2}$, do not forecast turbulence in the stratosphere.

Turbulence in the stratosphere should not be forecast if either of the following conditions occur: 1) The normal wind component at mountain-top level is at least 10 m sec^{-1}, and the minimum value of the average profile of L^2 exceeds $3 \times 10^{-7} m^{-2}$; 2) the normal wind component at mountain-top level is less than 10 m sec^{-1}, and the minimum value of the average profile of L^2 is less than $3 \times 10^{-7} m^{-2}$. In the first case, stability and wind characteristics in the middle and upper troposphere prevent extensive wave development. In the second case, the wind normal to the mountain tops is not strong enough to support development.

Cyclic variations with large amplitude in profiles of the ascent rate of rawinsonde balloons indicate the presence of mountain or lee waves, and hence can be used to determine if turbulence would be expected. When these features are observed and the amplitude of the cyclic variations exceeds about 50 m min^{-1}, turbulence should be expected in approximately 80 percent of the cases.

CONCLUDING REMARKS

From this investigation of turbulence encountered by the XB-70 airplane there seems to be sufficient evidence to support the hypothesis that there is a direct relationship between conditions in the troposphere favorable for mountain waves and the occurrence of turbulence in the stratosphere. This hypothesis does not explain all the occurrences of stratospheric turbulence; however, mountain waves do appear to be directly related to much of the turbulence in the stratosphere.

This conclusion is supported by the analytical and statistical findings of this research. The profiles of the Scorer parameter and Richardson number for mountain wave conditions agree well with the models and theoretical considerations. The shapes of the average profiles of these parameters are different for mountain-wave and no-mountain-wave conditions. Otherwise, if turbulence did occur in the stratosphere and the average profiles of the Scorer parameter and Richardson number did not resemble the profiles associated with mountain waves, the turbulence would have to be the result of an event other than mountain waves.

The value of the Richardson number is not a satisfactory forecast tool to delineate between conditions of turbulence and no turbulence. However, there are some definite statements which can be made. If the minimum value of the average profile of the Richardson number in the middle to upper troposphere exceeds 10, then turbulence in the stratosphere is unlikely. For average values of this parameter less than 10, other criteria are required to reach a conclusion.

The shape of the Richardson number profile helps to verify the presence of mountain-wave conditions. If the average profile of this parameter shows an increase in value from the mountain tops to the middle troposphere, a decrease from the middle to upper troposphere, and then a continuous increase into the stratosphere, it is compatible with the development of mountain waves. The minimum value in the middle or upper troposphere for such a profile should be less than 10.

The forecast methods developed showed about an 80-percent verification based either upon the normal wind component at mountain-top level and other features of the 700-mb level, the minimum value of the average profile of the Scorer parameter, or the presence of large amplitude cyclic variations in profiles of the ascent rate of rawinsonde balloons. For such a limited number of parameters, the results are encouraging.

The normal wind component of at least 10 m sec^{-1} at mountain-top level appears to be the most important of the parameters in the troposphere which were tested as indicators of the probable existence of turbulence in the stratosphere. Also, large horizontal gradients of at least 5 °C per 300 n mi for the 700- to 500-mb lapse rate of temperature were observed to be associated with the turbulence cases in the stratosphere. Gradients of 2 °C or less per 300 n mi showed a good correlation with the no-turbulence cases. However, the intermediate gradients of 3 and 4 °C per 300 n mi were equally representative of turbulence and no-turbulence cases.

The minimum profile value of the Scorer parameter is another important parameter. If this value is less than $3 \times 10^{-7} m^{-2}$ and the minimum wind component normal to the mountain crests is 10 m sec^{-1} or greater, extensive turbulence can be expected in the stratosphere. In addition, the shape and magnitude of the average profile of the Scorer parameter help to determine the existence of mountain waves.

Vertical wind shear is important in the formation of mountain waves and clear air turbulence. The effects of wind shear including its derivative as used in the second term of the Scorer parameter, are especially important from the mountain tops to the tropopause, and may be important at other altitudes.

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REFERENCES

1. Ehernberger, L. J.: Atmospheric Conditions Associated With Turbulence Encountered by the XB-70 Airplane Above 40,000 Feet Altitude. NASA TN D-4768, 1968.

2. Scoggins, J. R.: Relationships Between Synoptic Meteorological Parameters and Clear Air Turbulence Encountered by the XB-70 Airplane in the Stratosphere. NASA CR-106212, 1970.

3. Ehernberger, L. J.: Meteorological Aspects of High-Altitude Turbulence Encountered by the XB-70 Airplane. Proceedings of the Third National Conference on Aerospace Meteorology, Boston, American Meteorological Society, 1968.

4. Mitchell, F. A.; and Prophet, D. T.: Meteorological Analysis of Clear Air Turbulence in the Stratosphere. Clear Air Turbulence and Its Detection, Y. H. Pao and A. Goldburg, Eds., New York, Plenum Press, 1969, pp. 144-182.

5. Ashburn, E. V.; Waco, D. E.; and Melvin, C. A.: High Altitude Gust Criteria for Aircraft Design. AFFDL-TR-70-101, 1970.

6. Foltz, H. P.: Prediction of Clear Air Turbulence. Atmospheric Science Paper No. 106, Colorado State University, 1967.

7. Scorer, R. S.: Theory of Waves in the Lee of Mountains. Quart. J. Roy. Meteor. Soc., vol. 75, no. 323, 1949, pp. 41-56.

8. Scorer, R. S.: Theory of Airflow Over Mountains - III Airstream Characteristics. Quart. J. Roy. Meteor. Soc., vol. 80, no. 345, 1954, pp. 417-428.

9. Corby, G. A.; and Sawyer, J. S.: The Air Flow Over a Ridge - The Effects of the Upper Boundary and High Level Conditions. Quart. J. Roy. Meteor. Soc., vol. 84, no. 361, 1958, pp. 284-285.

10. Palm, E.; and Foldvik, A.: Contribution to the Theory of Two-Dimensional Mountain Waves. Geofys. Publikasjoner, vol. 21, no. 6, 1960.

11. Gazzola, A.: The Effects of Mountains on Air Currents (Translation). Redstone Scientific Information Center-131, U. S. Army Missile Command, Redstone Arsenal, Alabama, 1964.

12. Lumley, J. L.; and Panofsky, H. A.: The Structure of Atmospheric Turbulence. John Wiley & Sons, Inc., 1964.

13. Veazey, D. R.: A Literature Survey of Clear Air Turbulence. NASA CR-106211, 1970.

14. Axford, D. N.: An Observation of Gravity Waves in the Shear Flow in the Lower Stratosphere. Quart. J. Roy. Meteor. Soc., vol. 96, no. 408, pp. 273-286.

15. Scorer, R. S.: Mechanics of Clear Air Turbulence. Clear Air Turbulence and Its Detection, Y. H. Pao and A. Goldburg, Eds., New York, Plenum Press, 1969, pp. 34-50.

16. Badgley, F. I.: Large Scale Processes Contributing Energy to Clear Air Turbulence. Clear Air Turbulence and Its Detection, Y. H. Pao and A. Goldburg, Eds., New York, Plenum Press, 1969, pp. 109-126.

17. Hines, C. O.: Internal Atmospheric Gravity Waves at Ionospheric Heights. Can. J. Physics, vol. 38, no. 11, pp. 1441-1481.

18. Eliassen, A.; and Palm, E.: On the Transfer of Energy in Stationary Mountain Waves. Geofys. Publikasjoner, vol. 22, no. 3, 1961.

19. Booker, J. R.; and Bretherton, F. P.: The Critical Layer for Internal Gravity Waves in Shear Flow. J. Fluid Mech., vol. 27, part 3, 1967, pp. 513-539.

20. Hines, C. O.; and Reddy, C. A.: On the Propagation of Atmospheric Gravity Waves Through Regions of Wind Shear. J. Geophys. Res., vol. 72, no. 3, pp. 1015-1033.

21. Harrison, J. T.; and Sowa, D. F.: Mountain Wave Exposure on Jet Routes of Northwest Airlines and United Air Lines. UAL Meteorology Circular No. 60, Denver, 1966.

22. Harrison, H. T.: Forecasting the Mountain Wave at Denver, Colorado. UAL Meteorology Circular No. 42, Denver, 1957.

23. Harrison, H. T.: Progress in Forecasting High Level Clear Air Turbulence. UAL Meteorology Circular No. 52, Denver, 1961.

24. Corby, G. A.: A Preliminary Study of Atmospheric Waves Using Radiosonde Data. Quart. J. Roy. Meteor. Soc., vol. 83, no. 355, 1957, pp. 49-60.

25. Booker, D. Ray: Modification of Convective Storms by Lee Waves. Meteorological Monograph, vol. 5, no. 27, American Meteorology Society, 1963, pp. 129-140.

26. Hodge, Mary W.: Large Irregularities of Rawinsonde Ascensional Rates Within 100 Nautical Miles and Three Hours of Reported Clear Air Turbulence. Monthly Weather Review, vol. 95, no. 3, Mar. 1967, pp. 99-106.

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